Topic: Newton's method

Question: Use Newton's method to approximate to three decimal places the root of the function in the given interval.

$$x^2 - 1 = 0$$

on the interval $\left[\frac{1}{2}, 2\right]$

Answer choices:

A $x = 1.500$

B $x = 1.000$

C $x = 2.000$

D $x = 0.500$

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Solution: B

The root of a function is a point where the function crosses the x-axis. If you can graph the function to see the root, you'll be able to start your approximation closer to the actual answer, which will mean fewer applications of Newton's method.

If you can't graph the function, you can always use the left endpoint of the interval as your starting point.

No matter where you start, your answer using Newton's method will always be the same. It'll just take more applications of Newton's method if you start with an x_0 value that's farther away from the root.

The Newton's method formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

We'll pick a value for x_0 (either an approximation of the root based on the graph of the function, or the left endpoint of the given interval), and then use x_0 in the Newton's method formula to calculate x_1. We'll then use x_1 to find x_2, x_2 to find x_3, etc.

We'll keep going until we get two answers in a row that are the identical to the designated number of decimal places. This answer is then our root approximation using Newton's method.

Since we'll need it for our Newton's method formula, we'll start by taking the derivative of our function.

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$$f(x_n) = x_n^2 - 1$$

$$f'(x_n) = 2x_n$$

Now we can create our equation using

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{x_n^2 - 1}{2x_n}$$

Starting with the left endpoint of our interval, $1/2$, and work our problem with the number of decimals we were asked for, we get

$$x_0 = 0.500$$

$$x_1 = 0.500 - \frac{(0.500)^2 - 1}{2(0.500)} \qquad x_1 = 1.250$$

$$x_2 = 1.250 - \frac{(1.250)^2 - 1}{2(1.250)} \qquad x_2 = 1.025$$

$$x_3 = 1.025 - \frac{(1.025)^2 - 1}{2(1.025)} \qquad x_3 = 1.000$$

$$x_4 = 1.000 - \frac{(1.000)^2 - 1}{2(1.000)} \qquad x_4 = 1.000$$

Since our last two approximations were identical to three decimal places, we can stop and conclude that an approximation of the root of the given function in the given interval is $x = 1.000$.

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Topic: Newton's method

Question: Use Newton's method to approximate to three decimal places the root of the function in the given interval.

$$2x^2 - 3 = 0$$

on the interval $[1,2]$

Answer choices:

A $x = 1.525$

B $x = 1.255$

C $x = 1.522$

D $x = 1.225$

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Solution: D

The root of a function is a point where the function crosses the x-axis. If you can graph the function to see the root, you'll be able to start your approximation closer to the actual answer, which will mean fewer applications of Newton's method.

If you can't graph the function, you can always use the left endpoint of the interval as your starting point.

No matter where you start, your answer using Newton's method will always be the same. It'll just take more applications of Newton's method if you start with an x_0 value that's farther away from the root.

The Newton's method formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

We'll pick a value for x_0 (either an approximation of the root based on the graph of the function, or the left endpoint of the given interval), and then use x_0 in the Newton's method formula to calculate x_1. We'll then use x_1 to find x_2, x_2 to find x_3, etc.

We'll keep going until we get two answers in a row that are the identical to the designated number of decimal places. This answer is then our root approximation using Newton's method.

Since we'll need it for our Newton's method formula, we'll start by taking the derivative of our function.

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$$f(x_n) = 2x_n^2 - 3$$

$$f'(x_n) = 4x_n$$

Now we can create our equation using

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{2x_n^2 - 3}{4x_n}$$

Starting with the left endpoint of our interval, 1, and work our problem with the number of decimals we were asked for, we get

$$x_0 = 1.000$$

$$x_1 = 1.000 - \frac{2(1.000)^2 - 3}{4(1.000)} \qquad x_1 = 1.250$$

$$x_2 = 1.250 - \frac{2(1.250)^2 - 3}{4(1.250)} \qquad x_2 = 1.225$$

$$x_3 = 1.225 - \frac{2(1.225)^2 - 3}{4(1.225)} \qquad x_3 = 1.225$$

Since our last two approximations were identical to three decimal places, we can stop and conclude that an approximation of the root of the given function in the given interval is $x = 1.225$.

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Topic: Newton's method

Question: Use Newton's method to approximate to three decimal places the root of the function in the given interval.

$$x^2 - 3x - 1 = 0$$

on the interval $[3,4]$

Answer choices:

A $x = 3.303$

B $x = 3.322$

C $x = 3.032$

D $x = 3.332$

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Solution: A

The root of a function is a point where the function crosses the x-axis. If you can graph the function to see the root, you'll be able to start your approximation closer to the actual answer, which will mean fewer applications of Newton's method.

If you can't graph the function, you can always use the left endpoint of the interval as your starting point.

No matter where you start, your answer using Newton's method will always be the same. It'll just take more applications of Newton's method if you start with an x_0 value that's farther away from the root.

The Newton's method formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

We'll pick a value for x_0 (either an approximation of the root based on the graph of the function, or the left endpoint of the given interval), and then use x_0 in the Newton's method formula to calculate x_1. We'll then use x_1 to find x_2, x_2 to find x_3, etc.

We'll keep going until we get two answers in a row that are the identical to the designated number of decimal places. This answer is then our root approximation using Newton's method.

Since we'll need it for our Newton's method formula, we'll start by taking the derivative of our function.

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$$f(x_n) = x_n^2 - 3x_n - 1$$

$$f'(x_n) = 2x_n - 3$$

Now we can create our equation using

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{x_n^2 - 3x_n - 1}{2x_n - 3}$$

Starting with the left endpoint of our interval, 3, and work our problem with the number of decimals we were asked for, we get

$$x_0 = 3.000$$

$$x_1 = 3.000 - \frac{(3.000)^2 - 3(3.000) - 1}{2(3.000) - 3} \qquad x_1 = 3.333$$

$$x_2 = 3.333 - \frac{(3.333)^2 - 3(3.333) - 1}{2(3.333) - 3} \qquad x_2 = 3.303$$

$$x_3 = 3.303 - \frac{(3.303)^2 - 3(3.303) - 1}{2(3.303) - 3} \qquad x_3 = 3.303$$

Since our last two approximations were identical to three decimal places, we can stop and conclude that an approximation of the root of the given function in the given interval is $x = 3.303$.

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